Topics in Panel Data Econometrics

Supplement to Lecture 2

July 4, 2015

Embarrassing typos in page 1 of Lecture 2 The Ahn and Schmidt conditions should be

$$\mathbb{E}\left[(\alpha_i + \varepsilon_{it})\left(\varepsilon_{it-1} - \varepsilon_{i,t-2}\right)\right] = 0.$$

The notation in the assumption $\mathbb{E}[y_{i0}|\eta_i] = \eta_i/(1-\gamma)$ should be slightly changed to

$$\mathbb{E}[y_{i0}|\alpha_i] = \alpha_i/(1-\beta).$$

The random effects assumption should be $\mathbb{E}[u|X] = 0$ and $Var[u|X] = \Omega$, where $u_{it} = \alpha_i + \varepsilon_{it}$.

Mundlak-Chamberlain device, redux An alternative to the derivation in Lecture 2 can be found in Appendix 4 of Islam (1995, QJE). Moral-Benito (2013, JBES; 2014, JAE) extends likelihood methods to allow for predetermined regressors in the dynamic panel data model. In particular, he assumes that $\mathbb{E}\left[\varepsilon_{it}|y_i^{t-1}, x_i^t, \eta_i\right] = 0$, instead of the one I assume in Lecture 2, i.e. $\mathbb{E}\left[\varepsilon_{it}|y_i^{t-1}, x_i^T, \eta_i\right] = 0$. He augments the structural equation

$$y_{it} = \alpha y_{i,t-1} + \beta x_{it} + \eta_i + \epsilon_{it}$$

with reduced forms accounting for the predeterminedness of x_{iT} and initial conditions (y_{i0}, x_{i1}).

$$\begin{aligned}
x_{it} &= \gamma_{t0} y_{i0} + \cdots + \gamma_{t,t-1} y_{it-1} + \Lambda_{t1} x_{i1} + \cdots + \Lambda_{t,t-1} x_{it-1} + c_t \eta_i + \vartheta_{it}, \\
y_{i0} &= c_0 \eta_i + v_{i0}, \\
x_{i1} &= \gamma_{10} y_{i0} + c_1 \eta_i + \vartheta_{i1}.
\end{aligned}$$

Marginal effects Consider the following model where $Y_t = g(X_t, \alpha, \varepsilon_t)$. Correlated random effect models are really about reducing $X = (X_1, \ldots, X_T)$ to some function $V(X)$ of lower dimension. An object of interest for policy is how $E(Y_t|X = x)$ changes when x_s, the sth component of x, holding the source of unobserved heterogeneity constant. It is therefore of interest to determine how many time periods are need to identify this derivative. In particular,

$$
\begin{aligned}
E(Y_t|X=x) &= \int\int g(x_t,\alpha,\varepsilon) f_{\alpha,\varepsilon_t|X}(a,e|x)\, da\, de \\
&= \int\int g(x_t,\alpha,\varepsilon) f_{\alpha,\varepsilon_t|V(X)}(a,e|V(x))\, da\, de \\
&= \mu(x_t|X=x)
\end{aligned}
$$

and the derivative (by using the product rule) is given by

$$
\begin{aligned}
\frac{\partial E(Y_t|X=x)}{\partial x_s} &= \int\int \frac{\partial g(x_t,\alpha,\varepsilon)}{\partial x_s} f_{\alpha,\varepsilon_t|V(X)}(a,e|V(x))\, da\, de \\
&\quad + \int\int g(x_t,\alpha,\varepsilon) \frac{\partial f_{\alpha,\varepsilon_t|V(X)}(a,e|V(x))}{\partial V} \underbrace{\frac{\partial V(x)}{\partial x_s}}_{h_s(x)}\, da\, de \\
&= \frac{\partial \mu(x_t|X=x)}{\partial x_s} + c_t(x) h_s(x) \qquad (1)
\end{aligned}
$$

Note that the model implies that $\frac{\partial \mu(x_t|X=x)}{\partial x_s} = 0$ for all $t \neq s$. Try writing down the system of equations implied by (1) when $T=2$ and $T=3$. What do you notice? Can we identify these marginal effects?

What if a dimension-reducing function $V(x)$ is not available? Consider the case where $T=2$. It turns out that you can redo the calculations and will only be able to identify marginal effects only under time homogeneity

$$
f_{\alpha,\varepsilon_1|X}(a,e|x) = f_{\alpha,\varepsilon_2|X}(a,e|x)
$$

and conditioning on the set where $\{X_1 = X_2\}$.

What to read For more on the aspects of identification in fully nonparametric models, see the papers by Altonji and Matzkin (2005, Ecta), Bester and Hansen (2009, JBES), Hoderlein and White (2012, JoE), and Chernozhukov, Fernandez-Val, Hahn, and Newey (2013, Ecta). For the case where you have a fully parametric setup, life is considerably easier but one has to completely specify all aspects of the model including the distribution of unobserved heterogeneity. See Wooldridge (2005, JAE) and cited references for more.