Topics in Econometrics: Identification

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Recap

1. Defining identification – What can learned about some parameter from observables?
2. Ingredients – How were the data collected? How were the data generated?
3. A lot of examples showing restrictions that are needed to obtain identification – mostly rank-type conditions or subject-matter considerations
4. The by-products of identification arguments – give an understanding of identification failure; understand the extent of subject-matter knowledge that is required; which assumptions matter and which don't; a starting point to construct estimators
5. We now continue with more examples and move to more general settings.

1. (MA processes) Let $X_t = \mu + b_0\epsilon_t + b_1\epsilon_{t-1} + \ldots + b_p\epsilon_{t-p}$. Assume for the moment that $\epsilon_t \sim \text{iid } N(0, \sigma^2)$. We observe X_t but not the ϵ's. Can we identify the b's and μ? Take the case where $p = 1$.
2. (Simultaneous equations) Let

$$\begin{aligned} Y_1 &= \beta Y_2 + \gamma Z + \epsilon_1 \\ Y_2 &= \delta Y_1 + \epsilon_2 \end{aligned}$$

Suppose $\mathbb{E}(\epsilon_1 Z) = \mathbb{E}(\epsilon_2 Z) = 0$. Can we identify δ? How about β?

3. (Panel binary choice) Suppose (Y_1, Y_2, Y_3, α) is a random vector drawn independently such that

$$\begin{aligned} \Pr(Y_1 = 1|\alpha) &= p_1(\alpha) \\ \Pr(Y_2 = 1|\alpha, Y_1) &= F(\alpha + \gamma Y_1) \\ \Pr(Y_3 = 1|\alpha, Y_1, Y_2) &= F(\alpha + \gamma Y_2) \end{aligned}$$

where p_1 and F are both unknown functions with range in $[0, 1]$. In addition, F is strictly increasing.

The search ain't over III

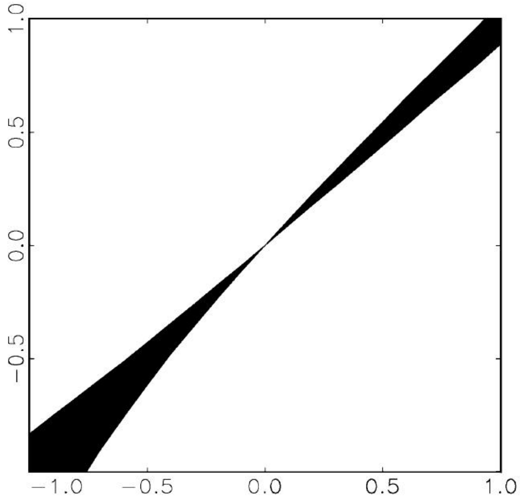


FIGURE 1.—Identified region for γ as a function of its true value.

4. (Dynamic panel data) Arellano and Bond (1991) proposed linear moment conditions of the form

$$\mathbb{E}[y_{is}(\Delta y_{it} - \alpha \Delta y_{i,t-1})] = 0, \ s = 0, \ldots, t-2; t = 2, \ldots, T,$$

   to identify α_0 in the dynamic panel data model $y_{it} = \alpha_0 y_{i,t-1} + c_i + v_{it}$ under the following assumptions:
   - $\mathbb{E}(v_{it}) = 0, \ t = 1, \ldots, T$
   - $\mathbb{E}(v_{it} y_{i0}) = 0, \ t = 1, \ldots, T$
   - $\mathbb{E}(v_{it} c_i) = 0, \ t = 1, \ldots, T$
   - $\mathbb{E}(v_{is} v_{it}) = 0, \ s = 1, \ldots, t-1; t = 1, \ldots, T$

   When will these linear moment conditions fail to identify α_0?
5. (Dynamic panel data) What if one uses Ahn-Schmidt quadratic moment conditions?

$$\mathbb{E}[(y_{iT} - y_{i,T-1}\alpha)(\Delta y_{it} - \alpha \Delta y_{i,t-1})] = 0, \ t = 2, \ldots, T$$

Let $R_n : \Omega \times B \to \mathbb{R}$ and $\overline{R}_n : B \to \mathbb{R}$ such that a.s.

$$\sup_B |R_n(\omega, \beta) - \overline{R}_n(\beta)| \to 0 \text{ as } n \to \infty.$$

Let $\overline{\beta}_n$ be an identifiably unique sequence of minimizers of $\overline{R}_n(\beta)$, then for any sequence $\widehat{\beta_n}$ such that eventually

$$R_n(\omega, \widehat{\beta_n}) = \inf_B R_n(\omega, \beta)$$

holds, we have $\rho_B(\widehat{\beta_n}, \overline{\beta}_n) \to 0$ a.s. as $n \to \infty$.

Remarks

1. If $\overline{R}_n$ does not depend on n and the parameter space B is compact, then identifiable uniqueness is equivalent to the existence of a unique minimizer of $\overline{R}$.
2. This only happens under correct specification.
3. Under misspecification, we might have a set of minimizers.
4. A reparametrization may be possible so that the lack of identification disappears.
5. See Section 4.6 of Pötscher and Prucha (1997) for more discussion of the identification conditions under misspecification.

1. Observe that if there exists a consistent estimator for a parameter θ, then the parameter is identifiable.
2. Gabrielsen (1978, JoE) gives a counterexample to show that the converse is not true.
3. Consider the model where $Y_i = \beta\rho^i + \epsilon_i$, $i = 1 \ldots, n$. Assume that ρ is known with $\rho < 1$, $\epsilon_i \sim \text{iid } N(0, 1)$ and $\beta > 0$.
4. Note that β is identifiable from the first moment of Y_i.
5. Form the ML estimator for β and show that it is not consistent.

A different type of argument: Measurement error model

Consider the model:

$$\begin{aligned} Y_1 &= \eta_1 + \epsilon_1 \\ Y_2 &= \eta_2 + \epsilon_2 \end{aligned}$$

where

1. $\eta_2 = \alpha + \beta\eta_1$
2. $\eta_1 \perp (\epsilon_1, \epsilon_2)$
3. (ϵ_1, ϵ_2) is bivariate normal with mean $(0, 0)$ and covariance matrix Σ
4. Y_1 and Y_2 are observable, but η_1 and η_2 are unobservable

A classic result: β is identifiable iff (η_1, η_2) is not bivariate normal.

1. Aim is to distinguish among endogenous effects, exogenous (contextual) effects, and correlated effects
2. Random sample (y, z, x, u) but only observe (y, x, z); y scalar outcome, x attributes characterizing an individual's reference group, (z, u) attributes that directly affect y
3. Model
$$y = \alpha + \beta \mathbb{E}(y|x) + \mathbb{E}(z|x)'\gamma + z'\eta + u$$
where $\mathbb{E}(u|x, z) = x'\delta$ and $(\alpha, \beta, \gamma, \delta, \eta)$ is parameter of interest.
4. Note that
$$\mathbb{E}(y|x, z) = \alpha + \beta \mathbb{E}(y|x) + \mathbb{E}(z|x)'\gamma + z'\eta + x'\delta$$

5. Find the social equilibrium:

$$\mathbb{E}(y|x) = \frac{\alpha}{1-\beta} + \mathbb{E}(z|x)' \left(\frac{\gamma+\eta}{1-\beta}\right) + x' \left(\frac{\delta}{1-\beta}\right)$$

6. But a reduced form model can be written as

$$\mathbb{E}(y|x,z) = \frac{\alpha}{1-\beta} + \mathbb{E}(z|x)' \left(\frac{\gamma+\beta\eta}{1-\beta}\right) + x' \left(\frac{\delta}{1-\beta}\right) + z'\eta$$

7. What parameters are identified?
8. When will the ability to detect a "social effect" break down?
9. What happens if $\delta = \gamma = 0$?

1. Random sample (y, x, v, v_0, v_1); Observe (y, x, v_0, v_1) only; Assume that $v_0 \leq v \leq v_1$ and all variables are scalar except for x.
2. What can we learn about $\mathbb{E}(v|x)$ and $\mathbb{E}(y|x, v)$?
3. Make the following assumptions:
   - I (Interval) $\Pr(v_0 \leq v \leq v_1) = 1$.
   - M (Monotonicity) $\mathbb{E}(y|x, v)$ exists and is weakly increasing in v.
   - MI (Mean independence) $\mathbb{E}(y|x, v, v_0, v_1) = \mathbb{E}(y|x, v)$
4. Under assumption I,

$$\mathbb{E}(v_0|x) \leq \mathbb{E}(v|x) \leq \mathbb{E}(v_1|x)$$

5. Under assumption IMMI,

$$\sup_{v_1 \leq V} \mathbb{E}(y|x, v_0, v_1) \leq \mathbb{E}(y|x, v = V) \leq \inf_{v_0 \geq V} \mathbb{E}(y|x, v_0, v_1).$$

Regressions with interval data II

6. Let $\mathbb{E}(y|x,v) = \Pr(y=1|x,v) = \Pr(x\beta + \delta v + \epsilon > 0|x,v)$.
7. Make additional assumptions for semiparametric identification:

SBR-1 For $\alpha \in (0,1)$, $\Pr(\epsilon \leq 0|x,v) = \alpha$.

SBR-2 $\epsilon \perp (v_0, v_1)|(x,v)$

SBR-3 $\delta > 0$

8. Let Assumption I and Assumptions SBR-1 to SBR-3 hold. Let $b \in \mathbb{R}^K$. Define

$$T(b) = \{(x, v_0, v_1) : \; xb + v_1 \leq 0 < x\beta + v_0\} \\ \cup \{(x, v_0, v_1) : \; x\beta + v_1 \leq 0 < xb + v_0\}.$$

The β is identified relative to b iff $\Pr(T(b)) > 0$.

Regressions with interval data III

9. Let $B^* = \{b \in \mathbb{R}^K : \Pr(T(b)) = 0\}$. Assume that
   - There exists no proper linear subspace of $\mathbb{R}^K$ having probability one under $F(x)$.
   - $\Pr(a_0 \leq v_0 \leq v_1 \leq a_1 | x) > 0$ for all $(a_0, a_1) \in \mathbb{R}^2$ such that $a_0 < a_1$, a.e. x.

   Then $B^* = \{\beta\}$.

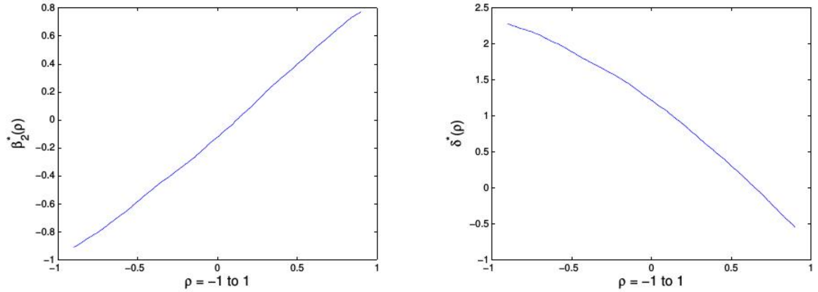
	Logit ML without Imputations (Table IV) Point Estimate (Confidence Region)	Logit MMD Interval Estimate (Confidence Region)	MMS Interval Estimate (Confidence Region)
Wealth	1.47	[1.21, 1.23]	[1.67, 1.68]
	(1.05, 1.89)	(.89, 1.62)	(.27, 4.6)
Age	.041	[.0412, .042]	[.044, .050]
	(.026, .056)	(.0352, .053)	(.014, .081)
Schooling	.053	[.033, .034]	[.019, .024]
	(.031, .076)	(.017, .058)	(.003, .054)
Body Mass	.064	[.067, .0671]	[.066, .067]
	(.05, .078)	(.055, .0785)	(.029, .095)
Constant	−4.17	[−4.28, −3.96]	−4
	(−5.26, −3.09)	(−5.58, −2.86)	

1. Consider the model

$$\begin{aligned} Y_1 &= 1(\beta_1 + u_1 > 0) \\ Y_2 &= 1(\beta_2 + \delta Y_1 + u_2 > 0) \end{aligned}$$

2. (u_1, u_2) is iid bivariate normal with mean $(0, 0)$, $var(u_1) = var(u_2) = 1$, and $cov(u_1, u_2) = \rho \in (-1, 1)$
3. (Y_1, Y_2) is observable.
4. What can be identified in this case?
5. What if we have

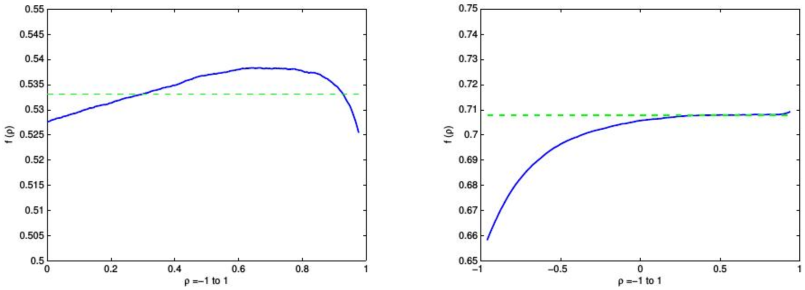
$$\begin{aligned} Y_1 &= 1(\beta_{11} + \beta_{12}x + u_1 > 0) \\ Y_2 &= 1(\beta_{21} + \beta_{22}x + \delta Y_1 + u_2 > 0) \end{aligned}$$



(a) (ρ, β_2) region

(b) (ρ, δ) region

Bivariate probit III



(a) $(\rho, f(\rho))$ region, $\beta_{21} = -0.4$, $\rho_0 = -0.3$ (b) (ρ, δ) region, $\beta_{21} = 0.4$, $\rho_0 = 0.5$

Simultaneous equations without exclusion restrictions

1. Consider the following demand-supply system:

$$\begin{aligned} Q_t &= \alpha + \beta P_t + \epsilon_t \\ Q_t &= \gamma + \theta P_t + u_t \end{aligned}$$

2. Assume (ϵ_t, u_t) is iid bivariate normal with zero means, variances σ_ϵ^2 and σ_u^2, and zero covariance.
3. As a consequence,

$$\begin{aligned} \mathbb{E}(P_t, Q_t) &= (\alpha - \gamma, \alpha\theta - \gamma\beta)/(\theta - \beta) \\ Var(P_t, Q_t) &= \begin{pmatrix} \sigma_\epsilon^2 + \sigma_u^2 & \theta\sigma_\epsilon^2 + \beta\sigma_u^2 \\ \theta\sigma_\epsilon^2 + \beta\sigma_u^2 & \theta^2\sigma_\epsilon^2 + \beta^2\sigma_u^2 \end{pmatrix} (\theta - \beta)^{-2} \end{aligned}$$

4. Is there any information in these moments that can help in identifying all the parameters of the model?